10/531143
JC12 Rec'd PCT/PTC 11 APR 2005

METHOD AND APPARATUS FOR DETERMINATION OF AN ACOUSTIC RECEIVER'S POSITION

1. FIELD OF THE INVENTION

The present invention pertains to determining the position of an acoustic receiver and, more particularly, to determining the position in an apparatus for seismic surveying.

2. DESCRIPTION OF THE RELATED ART

Seismic exploration is conducted on both land and in water. In both environments, exploration involves surveying subterranean geological formations for hydrocarbon deposits. A survey typically involves deploying acoustic source(s) and acoustic sensors at predetermined locations. The source(s) imparts acoustic waves into the geological formations. Features of the geological formation reflect the acoustic waves to the sensors. The sensors receive the reflected waves, which are the processed to generate seismic data. Analysis of the seismic data then indicates probable locations of the hydrocarbon deposits.

Accurate knowledge of source and sensor positions is important to the accuracy of the analysis. In land surveys, accurate positioning is not particularly difficult because environmental conditions are usually relatively stable. Sources and sensors can be readily positioned where desired and, once placed, they usually do not shift to any great degree. Marine surveys, however, are different altogether.

Marine surveys come in at least two types. In a first, an array of streamers and sources is towed behind a survey vessel. In a second type, an array of seismic cables, each of which includes multiple sensors, is laid on the ocean floor, or sea bottom, and a source is towed from a survey vessel. In both cases, many factors complicate determining the position of the sensors, including wind, currents, water depth, and inaccessibility.

In the second type of marine survey, where the array of seismic cables is laid on the sea floor, much attention is paid to the positioning of the seismic cables as they are laid. One important consideration is the shape of the seismic cables as they are deployed. The catenary

shape, or the shape of the seismic cable in the water during deployment, of a seismic cable must be known, or projected, if it is to be controlled effectively during deployment. Control is needed to optimize the deployment speed and accuracy. Control is also desired to avoid tangling the seismic cable with other obstructions, such as other cables or sub-sea devices. Remedial action can be taken to avoid such problems and improve the safety of sub-sea operations.

Current techniques apply various modeling techniques to project the shape and/or position of the seismic cable during deployment. These models consider the physical characteristics of the seismic cable (*e.g.*, weight, diameter, *etc.*) and account for the effect of predicted sea currents on the seismic cable as it descends to the sea floor. However, such methods provide only a model, or projection, of the seismic cable's shape and are predicated on a limited knowledge of the sea's properties.

The present invention is directed to resolving, or at least reducing, one or all of the problems mentioned above.

SUMMARY OF THE INVENTION

The invention comprises, in its various embodiments and aspects, a method and apparatus for determining a position of an acoustic receiver. The method includes determining a plurality of acoustic ranges from at least a first signal source position and a second signal source position, respectively, to the acoustic receiver; ascertaining a non-acoustic constraint on the acoustic receiver's position; and determining the acoustic receiver's position from the first and second acoustic ranges and the non-acoustic constraint. The apparatus includes at least one acoustic source; an acoustic receiver, and a computing system. The acoustic receiver is capable of receiving a plurality of acoustic signals transmitted by the at least one acoustic source from at least two signal source positions. The computing system is programmed to determine a position of the acoustic receiver from the acoustic ranges between the at least two signal source positions and the acoustic receiver and a non-acoustic constraint.

BRIEF DESCRIPTION OF THE DRAWINGS

The invention may be understood by reference to the following description taken in conjunction with the accompanying drawings, in which like reference numerals identify like elements, and in which:

FIG. 1 depicts the deployment of a subsea seismic cable in accordance with one particular embodiment of the present invention showing a representation of a cable catenary measurement using a combination of acoustic and angular measurements;

FIG. 2 illustrates, in a partially cut away view, one sensor module of the seismic cable in **FIG. 1**;

FIG. 3 represents a method practiced in accordance with the present invention to determine the position of an acoustic receiver;

FIG. 4 and **FIG. 5** conceptually illustrate the analytical determination of the position of a sensor module as deployed in **FIG. 1** in accordance with a first embodiment of the present invention;

FIG. 6 represents one particular implementation of the method in **FIG. 3** wherein the positions of points on the cable are determined analytically as illustrated in **FIG. 4** and **FIG. 5**;

FIG. 7 represents a one particular implementation of the method in **FIG. 3** alternative to that of **FIG. 4** – **FIG. 6** wherein the position is determined using a model-based solution;

FIG. 8 is a block diagram depicting a computing system such as may be used in implementing certain aspects of the present invention;

FIG. 9 represents a method by which the present invention may for dynamically determining the shape of a body in accordance with a first implementation of one application of the present invention;

FIG. 10 represents a method by which the shape of a cable may be determined dynamically practiced in accordance with a second implementation of the application;

FIG. 11 represents a method wherein the positions of points on the cable are determined through application of an inversion algorithm;

FIG. 12 projects the cable position onto the x-y plane (top) and x-z plane (bottom) for a single acoustic source experiment simulating an application of the invention to determine the shape of a seismic cable as it is deployed;

FIG. 13 graphs the cable position error for the single acoustic source experiment of **FIG. 12**;

FIG. 14 projects the cable position onto the *x-y* plane (top) and *x-z* plane (bottom) for a three acoustic source experiment simulating an application of the invention to determine the shape of a seismic cable as it is deployed;

FIG. 15 graphs the cable position error for the three acoustic source experiment in **FIG. 14**;

FIG. 16 projects the cable position onto the *x-y* plane (top) and *x-z* plane (bottom) for a three source and dip angle experiment simulating an application of the invention to determine the shape of a seismic cable as it is deployed; and

FIG. 17 graphs the cable position error for the three source experiment and dip experiment in **FIG. 16**.

While the invention is susceptible to various modifications and alternative forms, the drawings illustrate specific embodiments herein described in detail by way of example. It should be understood, however, that the description herein of specific embodiments is not intended to limit the invention to the particular forms disclosed, but on the contrary, the intention is to cover all modifications, equivalents, and alternatives as defined by the appended claims.

DETAILED DESCRIPTION OF THE INVENTION

Illustrative embodiments of the invention are described below. In the interest of clarity, not all features of an actual implementation are described in this specification. It will of course be appreciated that in the development of any such actual embodiment, numerous implementation-specific decisions must be made to achieve the developers' specific goals, such as compliance with system-related and business-related constraints, which will vary from one implementation to another. Moreover, it will be appreciated that such a development effort, even if complex and time-consuming, would be a routine undertaking for those of ordinary skill in the art having the benefit of this disclosure.

FIG. 1 conceptually illustrates a seismic cable 100 being deployed from a vessel 105 that is also towing a first source 110 and a second source 115. The seismic cable 100 comprises a plurality of sensor modules 120 – 126 on a cable 130 and terminating in a steerable anchor 145. The sensor modules 120 – 126 are, in the illustrated embodiment, evenly spaced apart a known, predetermined distance, d_r, along the seismic cable 100. As is

shown in **FIG. 2** for a sensor module 200, each sensor module includes an acoustic receiver 205 and contains, in the illustrated embodiment, an angular orientation measurement device 210 in a housing 215. The acoustic receivers 205 and angular orientation measurement devices 210 receive power over leads 220 through the cable 130. The sensor modules 120 – 126 may also include additional acoustic receivers (not shown) for use in a seismic survey that may be either hydrophones (not shown) or geophones (also not shown).

Note that, in alternative embodiments, the sensor modules 120 – 126 may be spaced apart at irregular distances or clustered on the seismic cable 100. Note also that, in alternative embodiments, the acoustic and angular orientation measurement devices may be housed separately. The angular orientation measurement devices 210 may be, *e.g.*, inclinometers, but other types may be used. The sensors 120 – 126 may also house other kinds of sensors, *e.g.*, magnetometers or compasses for measuring heading. The steerable anchor 132 may be controlled from the surface 150 of the sea 155 (shown in **FIG. 1**) in conventional fashion as the seismic cable 100 descends to the sea floor 160 to help position the seismic cable 100 as desired.

The scenario depicted in **FIG. 1** is illustrative only. For example, the acoustic sources 110, 120 may be towed from vessels other than the vessel 105 from which the seismic cable 100 is deployed. The acoustic sources 110, 115 may also be deployed from separate vessels in some embodiments or from buoys. Similarly, the seismic cable 100 is shown with seven sensor modules 120 – 126, even though the invention is not limited by the number of such sensor modules. Indeed, the invention may be applied to embodiments in which the seismic cable, *e.g.*, the seismic cable 100, includes any number of sensor modules, *e.g.*, the sensor modules 120 – 126. Those skilled in the art having the benefit of this disclosure will nevertheless appreciate that certain implementation-specific, practical considerations will circumscribe the range of the number of sensor modules.

As the seismic cable 100 is deployed, environmental conditions, such as wind and current, impart forces on the seismic cable 100 and the vessel 105. These forces distort the path of the seismic cable 100 along all three coordinate axes x, y, and z, causing deviations in the path. These deviations, in turn, affect the position of the sensor modules 120 – 126—and, hence, the position of any sensors and/or receivers they house—on the sea floor 160. The state of the art provides numerous techniques by which these deviations can be projected or

predicted, but none by which they can be empirically determined. Thus, the state of the art fails to provide a technique by which the position of the sensor modules 120 – 126 can be known. The present invention, however, remedies this deficiency.

FIG. 3 represents a method 300 for determining a position of an acoustic receiver, *e.g.*, an acoustic receiver 305, in accordance with the present invention. The method begins by first determining (at 310) a plurality of acoustic ranges from at least a first signal source position (*e.g.*, the position of the first acoustic source 110) and a second signal source position (*e.g.*, the position of the second acoustic source 115), respectively, to the acoustic receiver 305. A non-acoustic constraint (*e.g.*, the angular orientation of the sensor module 121) on the acoustic receiver's position is also ascertained (at 320). The acoustic receiver's position is then determined (at 330) from the first and second acoustic ranges and the non-acoustic constraint.

The invention admits variation in the practice of the method 300. For instance, the non-acoustic constraint can be ascertained (at 320) prior to determining the acoustic ranges (at 310). A wide variety of non-acoustic constraints may also be employed. In the illustrated embodiment, the non-acoustic constraint is the respective angular orientation of the sensor modules 120 – 126 as measured by the respective angular orientation measurement devices 210. However, as will become apparent from the discussion below, other non-acoustic constraints may also be employed. The determination of the acoustic receiver's position may also be implemented in various ways. Two such implementations are discussed in further detail below. The present invention may also find many applications. The illustrated embodiment employs the invention not only to determine the positions of the acoustic receivers, but to also determine the shape of the seismic cable 100. In one implementation, the shape of the seismic cable 100 is determined dynamically as the seismic cable 100 is deployed.

The determination of the acoustic ranges (at 320, in **FIG. 3**) can be implemented in a variety of ways. Two alternative embodiments are discussed further below. The first is an analytical-based solution illustrated in **FIG. 4** – **FIG. 6**. This analytical determination employs the analysis represented by Eq. (1) – Eq. (11) and the discussion that follows. The second embodiment is a model-based approach illustrated in **FIG. 7** and employing the analysis represented by Eq. (1) – Eq. (3) and Eq. (12) – Eq. (22). This approach applies an

inversion algorithm to correct positions initially determined by a conventional model. The inversion algorithm iteratively applies a non-linear inversion represent by Eq. (22) to the predicted positions using the dynamic angular orientations. However, the invention admits wide variation in performing the determination (at 620). Additional alternative embodiments may become apparent to those in the art having the benefit of this disclosure.

Referring again to **FIG. 1**, as the seismic cable 100 is deployed, the acoustic sources 110, 115 emit acoustic signals 165. Note that, in a seismic cable 100 including N sensor modules 120 - 126, there are 3 x N degrees of freedom. The seismic cable 100 enters the sea 155, in the illustrated embodiment, at $x = y = z = 0$. Note that the origin of the x-y-z coordinate system is arbitrarily selected, and other points may be used upon accounting for any translation. The sensor modules 120 – 126 receive the acoustic signals 165 from which they determine the acoustic range of the respective acoustic receiver 205 to the acoustic sources 110, 115.

In a first embodiment the acoustic receiver's position is determined analytically. In determining the acoustic ranges, the one way travel time t_i for an acoustic signal 165 emitted from one of the high frequency, acoustic sources 110, 115 at position (x_s, y_s, z_s) to each sensor module 120 – 126 at position (x_i, y_i, z_i) is given by:

$$(x_i - x_s)^2 + (y_i - y_s)^2 + (z_i - z_s)^2 = v_0^2 t_i^2 \tag{1}$$

Eq. (1) can be repeated for different acoustic sources, and, thus, for each of the acoustic sources 110, 115. Note that Eq. (1) constrains the position of the sensor modules 120 – 126 to within a sphere centered on (x_i, y_i, z_i) with radius a v_t. The distance d_r between the sensor modules 120 – 126 is fixed (*e.g.*, 12.5 m, in one particular embodiment) and can be approximated, assuming limited curvature of the cable 130 between sensors, by:

$$[x_{i+1} - x_i]^2 + [y_{i+1} - y_i]^2 + [z_{i+1} - z_i]^2 = dr^2 \tag{2}$$

The inline angle of the sensor housing 215, or angular orientation of the acoustic receiver 205, with the horizontal plane α_i is a derivative of the position vector. This can be represented by:

$$\frac{(z_{i+1}-z_i)^2}{(x_{i+1}-x_i)^2+(y_{i+1}-y_i)^2}=\tan^2\alpha_i \qquad (3)$$

Eq. (2) and Eq. (3) can then be combined to constrain the positions of the sensor modules 120 – 126 to a plane.

Various other measurements can also contribute to the solution in addition to, or in lieu of, the angular orientation: the water depth of the sensor modules 120 – 126; the distance between the vessel 105 and the position of the closest sensor; a depth measurement derived from the hydrostatic pressure; measurements taken with a second acoustic source 115; and magnetometers (not shown) inside the seismic cable 100. Note that these are all non-acoustic constraints. In embodiments employing these other non-acoustic constraints, the sensor modules 120 – 126 will contain appropriate sensors in addition to, or in lieu of, the angular orientation measurement devices 210. In one particular embodiment, the water depth of the position of the acoustic receiver 120 – 126 is determined. One implementation of this particular embodiment measures the depth with a pressure sensor. A second implementation, however, retrieves the information from a data archive depending on other sensed data. Thus, the ascertainment of the non-acoustic constraint may include, *e.g.*, retrieving an archived water depth measurement for the acoustic receiver's position.

Note that, when an acoustic source (*e.g.*, acoustic sources 110, 115) is located on the vessel 105, it will be inline with the cable direction, but the cross-line direction is poorly constrained. Only a second acoustic source offset from the inline direction and angular orientation measurement devices 210 inside the seismic cable 100 can resolve this cross-line issue. Thus, in practice, the invention employs at least the two acoustic sources. Alternative embodiments, however, may use three or more acoustic sources to facilitate the position determination. Theoretically, however, a single source may be employed in some embodiments to generate separate acoustic signals at two different signal source positions from which two different acoustic ranges may be obtained.

Thus, the position of each sensor module 120 – 126 can be determined analytically from their angular orientations constrained by, *e.g.*, acoustic travel times. More particularly, the positions can be computed as the intersection of two spheres and a plane. **FIG. 4** conceptually illustrates the analytical solution 400 which is the intersection 405 of two

spheres 410, 415 and a plane 420. Referring to both **FIG. 1** and **FIG. 4**, the travel times from the two acoustic sources 110, 115 to a given one of the sensor modules 120 - 126 define the two spheres 410, 415 and the plane 420 represents the dip measurement resulting from the angular orientation of the respective sensor module 120 - 126. Each of the sensor modules 120 - 126 is separated by the distance d_r on the seismic cable 100, as was mentioned above.

Generically, there are two points of intersection 405, shown in **FIG. 5**. In practice, using a third acoustic source 165 (shown in **FIG. 1**), this can be reduced to one point of intersection 405. Thus, the position of each sensor module 120 – 126 can be uniquely determined. To analytically determine the position for any one sensor module 120 – 126, the position of one acoustic source 110, 115 is temporarily assigned to be the origin. Each of the spheres 410, 415 is then represented by:

$$(x,x) = r^2 \tag{4}$$

and:

$$(x-m,x-m) = s^2 \tag{5}$$

where **m** is the location of the second acoustic source 110, 115 relative to the first acoustic source 110, 115 and **x** is the position of the sensor module 120 – 126.

The equation for the plane 420 is:

$$(\mathbf{x},\mathbf{n}) = d \tag{6}$$

where **n** = (0,0,1). The intersection of the two spheres 410, 415 is a circle 500, shown in **FIG. 5**. This circle 500 is confined to a plane given by:

$$(x,m) = ((m,m)+r^2-s^2)/2 \tag{7}$$

The intersection of this plane with the 'dip' plane 420 is a line 420 with a parameterization **a**+λ(**nxm**). Define **a** to be a linear combination of **n** and **m**:

$$\mathbf{a}=x\mathbf{n}+y\mathbf{m} \tag{8}$$

where

$$x = \frac{(n,n)r-(n,m)s}{(n,n)(m,m)-(n,m)^2} \tag{9}$$

and

$$y = \frac{(n,n)s-(n,m)r}{(n,n)(m,m)-(n,m)^2} \tag{10}$$

The points of intersection of the line 420 with the first sphere 410 gives us two values for λ:

$$\lambda_{1,2} = \pm\sqrt{(r^2 - (a,a))/(v,v)}. \tag{11}$$

The two possible positions of the hydrophone now are $a + \lambda_1 v$, and $a + \lambda_2 v$, represented by the points 405 in **FIG. 5**.

Using the measurement from the third acoustic source 165, or another constraint, one of the locations (*i.e.*, intersections 405) can be selected that corresponds to the true position of the sensor module 120 – 126. Having determined the position of the first sensor module 120 – 126, the location of the second sensor module 120 – 126 can be determined in like manner, and so on. Eventually, the position of each sensor module 120 – 126 can be determined. The manner in which this additional constraint is applied will depend, to some degree, on the constraint itself. For instance, in one embodiment, one intersection 405 can be eliminated as an improbable physical location because it is, *e.g.*, above the surface 150 or ahead of the survey vessel 105.

The determination of the acoustic receiver's position (at 330, in **FIG. 3**) by the analytical technique discussed immediately above is represented in **FIG. 6**. The analytical determination includes first calculating (at 610) the intersection (*e.g.*, the intersection 405) of a first sphere (*e.g.*, the sphere 410), a second sphere (*e.g.*, the sphere 415), and a plane (*e.g.*, the plane 420). The first sphere and the second sphere are defined by the positions of a first acoustic source (*e.g.*, the acoustic source 110) and a second acoustic source (*e.g.*, the acoustic source 115), respectively, relative to the position of the respective sensor module 120 - 126. Next, the analytical determination (at 1010) selects (at 620) one point of the intersection to identify the position of the respective sensor module. The illustrated embodiment accomplishes this by applying another constraint, *e.g.*, acoustic travel time from a third acoustic source (*e.g.*, the acoustic source 125).

A second embodiment determines the position of the acoustic receiver through a model-based approach using an inversion algorithm. Typically, there are errors in the data (both the travel time and the dip measurements). These errors are not taken into account in the analytical determination of the position of the seismic cable 100. These errors are expected to propagate, at least to some degree, because of the iterative nature of the

illustrated embodiment. To determine the effect of these errors, this second embodiment applies a linear inversion method to correct for these errors.

More particularly, Eq. (1) – Eq. (3) are linearised by perturbing the position vector, as in Eq. (12) – Eq. (14) below:

$$x_i = x_i^{'} + dx_i \qquad (12)$$

$$y_i = y_i^{'} + dy_i \qquad (13)$$

$$z_i = z_i^{'} + dz_i \qquad (14)$$

Inserting Eq. (12) – Eq. (14) into Eq. (1), and dropping second order terms gives:

$$2(x_i - x_s)dx_i + 2(y_i - y_s)dy_i + 2(z_i - z_s)dz_i = v^2 t_i^2 - (x_i - x_s)^2 - (y_i - y_s)^2 - (z_i - z_s)^2 \qquad (15)$$

In a similar way, the substitution of Eq. (12) – Eq. (14) into Eq. (2) gives:

$$2[x_i - x_{i+1}]dx_i + 2[x_{i+1} - x_i]dx_{i+1} + 2[y_i - y_{i+1}]dy_i + 2[y_{i+1} - y_i]dy_{i+1} + 2[z_i - z_{i+1}]dz_i + 2[z_{i+1} - z_i]dz_{i+1} = dr^2 - [x_i - x_{i+1}]^2 - [y_i - y_{i+1}]^2 - [z_i - z_{i+1}]^2 \qquad (16)$$

Differentiation of Eq. (3) then gives Eq. (17):

$$Adx_i - Adx_{i+1} + Bdy_i - Bdy_{i+1} + Cdz_i - Cdz_{i+1} = \tan^2 \alpha_i - \frac{(z_{i+1} - z_i)^2}{(x_{i+1} - x_i)^2 + (y_{i+1} - y_i)^2} \qquad (17)$$

$$A = \frac{2(x_{i+1} - x_i)(z_{i+1} - z_i)^2}{\left[(x_{i+1} - x_i)^2 + (y_{i+1} - y_i)^2\right]^2} \qquad (18)$$

$$B = \frac{2(y_{i+1} - y_i)(z_{i+1} - z_i)^2}{\left[(x_{i+1} - x_i)^2 + (y_{i+1} - y_i)^2\right]^2} \qquad (19)$$

$$C = \frac{-2(z_{i+1} - z_i)}{\left[(x_{i+1} - x_i)^2 + (y_{i+1} - y_i)^2\right]} \qquad (20)$$

For two sensors and two acoustic sources these equations can be written as a linear matrix equation Eq. (21):

$$\begin{bmatrix} 2(x_i - x_{s1}) & 2(y_i - y_{s1}) & 2(z_i - z_{s1}) & 0 & 0 & 0 \\ 2(x_i - x_{s2}) & 2(y_i - y_{s2}) & 2(z_i - z_{s2}) & 0 & 0 & 0 \\ 0 & 0 & 0 & 2(x_{i+1} - x_{s1}) & 2(y_{i+1} - y_{s1}) & 2(z_{i+1} - z_{s1}) \\ 0 & 0 & 0 & 2(x_{i+1} - x_{s2}) & 2(y_{i+1} - y_{s2}) & 2(z_{i+1} - z_{s2}) \\ 2x_i - 2x_{i+1} & 2y_i - 2y_{i+1} & 2z_i - 2z_{i+1} & 2x_{i+1} - 2x_i & 2x_{i+1} - 2x_i & 2x_{i+1} - 2x_i \\ A & -A & B & -B & C & -C \end{bmatrix} \begin{bmatrix} dx_i \\ dy_i \\ dz_i \\ dx_{i+1} \\ dy_{i+1} \\ dz_{i+1} \end{bmatrix} =$$

$$\begin{bmatrix} v^2 t_i^2 - (x_i - x_{s1})^2 - (y_i - y_{s1})^2 - (z_i - z_{s1})^2 \\ v^2 t_i^2 - (x_i - x_{s2})^2 - (y_i - y_{s2})^2 - (z_i - z_{s2})^2 \\ v^2 t_{i+1}^2 - (x_{i+1} - x_{s1})^2 - (y_{i+1} - y_{s1})^2 - (z_{i+1} - z_{s1})^2 \\ v^2 t_{i+1}^2 - (x_{i+1} - x_{s2})^2 - (y_{i+1} - y_{s2})^2 - (z_{i+1} - z_{s2})^2 \\ dr^2 - [x_i - x_{i+1}]^2 - [y_i - y_{i+1}]^2 - [z_i - z_{i+1}]^2 \\ \tan^2 \alpha_i - \dfrac{(z_{i+1} - z_i)^2}{(x_{i+1} - x_i)^2 + (y_{i+1} - y_i)^2} \end{bmatrix} \qquad (21)$$

When three acoustic sources are used two additional lines similar two the top two lines are added.

A solution to the non-linear equations Eq. (1) – Eq. (3) is found, in the illustrated embodiment, using the iterative non-linear inversion technique taught by Tarantola, A. and Valette, B., "Generalized Non-linear Inverse Problems Solved Using the Least Squares Criterion," Rev. Geophys. Space Physics, 20, 219-232 (1982). This technique applies Eq. (9):

$$\mathbf{p}_{k+1} = \mathbf{p}_0 + C_{p_0 p_0} \cdot G_k^{\,T} \cdot \left(C_{d_0 d_0} + G_k \cdot C_{p_0 p_0} \cdot G_k^{\,T} \right)^{-1} \cdot \left[\mathbf{d}_0 - \mathbf{g}(\mathbf{p}_k) + G_k \cdot (\mathbf{p}_k - \mathbf{p}_0) \right] \qquad (22)$$

where:

$\mathbf{p}_0 \equiv$ the initial position vector of the cable;

$\mathbf{p}_k \equiv$ the position after the k^{th} iteration;

$C_{p_0 p_0} \equiv$ t he model covariance matrix;

$C_{d_0 d_0} \equiv$ the data covariance matrix;

$\mathbf{d}_0 \equiv$ the data;

$\mathbf{g} \equiv$ the forward model operator; and

$G_k \equiv$ the matrix containing the partial derivatives (*see* equation (23) of Tarantola and Valette (1982)).

The covariance and resolution matrix can also be computed. The term $\mathbf{d_0}$-$\mathbf{g}(\mathbf{p_k})$ in Eq. (22) is equal to the right-hand side of Eq. (21), G_k equal to the left-hand side matrix in Eq. (2), and $\mathbf{p_k}$ to the left-hand side vector.

Note that, although the illustrated embodiment employs the iterative non-linear inversion technique of Tarantola and Valette, other suitable techniques known to the art may be employed. Iterative non-linear inversion techniques are well known to the art and many are readily and commercially available in off-the-shelf computational software applications. For instance, the MatLab™ software package typically includes suitable techniques. Alternative embodiments might also employ other techniques, such as least-squares fit.

The determination of the acoustic receiver's position (at 330, in **FIG. 3**) by the model-based technique discussed immediately above is represented in **FIG. 7**. The method 700 begins by modeling (at 710) the acoustic receiver's position from historical positions associated with the acoustic receiver's position. Next, the method 700 applies (at 720) an inversion algorithm to constrain the modeled position with the non-acoustic constraint. The method 700 is then iterated as the position of the acoustic receiver changes over time.

Many aspects of the present invention are, in the illustrated embodiments, implemented in software, although the invention is not so limited. In alternative embodiments, these aspects may be implemented in electronic hardware or some combination of hardware and software. Nevertheless, some portions of the detailed descriptions herein are presented in terms of a software implemented process involving symbolic representations of operations on data bits within a memory in a computing system or a computing device. These descriptions and representations are the means used by those in the art to most effectively convey the substance of their work to others skilled in the art. The process and operation require physical manipulations of physical quantities. Usually, though not necessarily, these quantities take the form of electrical, magnetic, or optical signals capable of being stored, transferred, combined, compared, and otherwise manipulated. It has proven convenient at times, principally for reasons of common usage, to refer to these signals as bits, values, elements, symbols, characters, terms, numbers, or the like.

It should be borne in mind, however, that all of these and similar terms are to be associated with the appropriate physical quantities and are merely convenient labels applied to these quantities. Unless specifically stated or otherwise as may be apparent, throughout the present disclosure, these descriptions refer to the action and processes of an electronic device, that manipulates and transforms data represented as physical (electronic, magnetic, or optical) quantities within some electronic device's storage into other data similarly represented as physical quantities within the storage, or in transmission or display devices. Exemplary of the terms denoting such a description are, without limitation, the terms "processing," "computing," "calculating," "determining," "displaying," and the like.

Note also that the software implemented aspects of the invention are typically encoded on some form of program storage medium or implemented over some type of transmission medium. The program storage medium may be magnetic (*e.g.*, a floppy disk or a hard drive) or optical (*e.g.*, a compact disk read only memory, or "CD ROM"), and may be read only or random access. Similarly, the transmission medium may be twisted wire pairs, coaxial cable, optical fiber, or some other suitable transmission medium known to the art. The invention is not limited by these aspects of any given implementation.

The vessel 105 is therefore equipped with a rack-mounted computing apparatus 800, conceptually illustrated in **FIG. 8,** with which these aspects of the invention executed. The computing apparatus 800 includes a processor 805 communicating with some storage 810 over a bus system 815. The storage 810 may include a hard disk and/or RAM and/or removable storage such as a floppy magnetic disk 817 and an optical disk 820. The storage 810 is encoded with a data structure 825 storing the data set acquired as discussed above, an operating system 830, some user interface software 835, and an application 865. The user interface software 835, in conjunction with a display 840, implements a user interface 845. The user interface 845 may include peripheral I/O devices such as a keyboard 850, a mouse 855, or a joystick 860. The processor 805 runs under the control of the operating system 830, which may be practically any operating system known to the art.

The processor 805, under the control of the operating system 830, invokes the user interface software 835 on startup so that the operator can control the computing apparatus 800. The application 865 is invoked by the processor 805 under the control of the operating

system 830 or by the user through the user interface 845. The application 865, when executed by the processor 805, determines the shape of the cable either analytically, as in **FIG. 4** – **FIG. 6** or by the iterative, model based approach as in **FIG. 7**, depending on the embodiment implemented. The application 865 may also perform other functions, such as displaying the determined position.

The invention may find many applications in the context of seismic surveying. In the illustrated embodiment, for instance, the invention may be used to determine the position of each of the acoustic receivers 205 for each of the sensor modules 120 – 126 where they are deployed for a survey. As will be appreciated by those skilled in the art having the benefit of this disclosure, that knowledge is, in itself, useful in conducting a seismic survey. However, because multiple positions are known, and because they are constrained to the cable 130—in the illustrated embodiment—the shape of the seismic cable 100 can be determined. Furthermore, if the positions are determined dynamically during deployment, the shape of the seismic cable 100 can be used to steer it during the deployment to help position the seismic cable 100 where desired for the seismic survey.

For instance, **FIG. 9** illustrates a method 900 by which the present invention may be implemented for dynamically determining the shape of a body, *e.g.*, the seismic cable 100. In general terms, the sensor modules 120 – 126 take certain measurements from which the effects of the deviations during deployment can be determined. From the known deviations, the position of the sensor modules 120 – 126 can be determined. In turn, the shape of the seismic cable 100 can be determined from the known positions. More particularly, the method 900 begins by measuring the dynamic angular orientation of at least two points on the body (at 910). This may include, for example, measuring the inclination and roll at that point on the cable. Note that the phrase "dynamic angular orientation," as used herein, implies that the point is moving or is subject to movement. The method 900 then determines the position of the two points from the measured angular orientations dynamically (*i.e.*, on-the-fly) (at 920). Note here that the term "determination" encompasses a determination within some acceptable degree of error, as no such determination is without some degree of inaccuracy. The term "dynamically" implies in real-time or in near real-time.

In one embodiment, dynamically determining the position of the two points from the measured angular orientation (at 920, in **FIG. 9**) comprises a method illustrated in **FIG. 10**.

The method 1000 comprises first analytically determining (at 1010) the respective positions of a plurality of points (*e.g.*, the sensor modules 120 – 126) on a cable (*e.g.*, the seismic cable 100). In the context of the present invention, the phrase "analytically determined" means to calculate from actual measurements, as opposed to projected or predicted positions. One technique is discussed above relative to **FIG. 4** – **FIG. 6**. In the that embodiment, these measurements are of inclination, roll, acoustic travel time (or range), although some implementations further include, *e.g.*, water depth and/or heading measurements. From these analytically determined positions, the method 1000 determines (at 1020) the shape of the cable.

In another embodiment, as shown in **FIG. 11**, a model-based method 1100 is employed to determining the position of the two points from the measured angular orientation (at 920, in **FIG. 9**). The method 1100 is one implementation of the model-based technique previously discussed relative to **FIG. 7**. The method 1100 begins by modeling the respective positions of a plurality of acoustic receivers as the seismic cable 100 is deployed (at 1110). Any conventional modeling package known to the art suitable for this purpose may be used. The method 1100 then applies an inversion algorithm (at 1120) to constrain the modeled positions with the respective measured dynamic angular orientation of the respective positions. The non-linear inversion technique discussed above is applied at predetermined time intervals to include the uncertainties and update the cable position. The cable position (*i.e.*, the positions of the sensor modules 120 – 126) from the previous inversion is used as a starting point from which to find the small changes to determine the new position. Such a process minimizes the number of iterations in the inversion process and speeds the delivery of the solution. The full solution is nevertheless periodically fully determined (at 1130) as a check to ensure that systematic errors are not accumulating in the real-time solutions, and that the error function for the solution is indeed the global minimum. The non-linear inversion is then applied (at 1130) to this new analytically determined position.

A number of simulations were performed to quantify the value of adding additional measurements to the inversion for cable position. Each simulation was performed using a seismic cable including ten sensor modules, dipping from the horizontal at an angle of 22.50°, and with a sensor module spacing of $d_r = 12.5$ m. The position of the first sensor was (11.54, 0, -4.78). In all the simulations, the assumed initial position of the cable was determined analytically as described above. This initial position is exact in the case of zero

errors in the data. Errors in the data were accounted for by performing a non-linear inversion, as was discussed above. This non-linear inversion can be repeated until convergence is obtained. The new cable position estimate can be compared with the known exact final position of the cable to find the accuracy, or error, in the estimated position. These errors are presented for each simulation. Failure to include the uncertainties in the inversion process may lead to the introduction of systematic errors in the seismic cable's position estimate. The included uncertainties are listed in Table 1 below. These simulations show that including dip measurements can significantly reduce the error in position of the seismic cable.

Table 1. Measurement Uncertainties In Simulations

Parameter	Uncertainty
Sensor dip	0.5°
Water velocity	30m/s
Distance between sensors	0.05m

In a first simulation, only one acoustic source and no dip measurements were used. The source position was (-12.5, 0, 0). The system of linear equations is now underdetermined and regularized using the data and model covariance matrices. Plots of the cable position before and after inversion are shown in **FIG. 12**, which projects the cable position onto the *x-y* plane (top) and *x-z* plane (bottom) for this simulation. The true cable position is represented by the curves 1200, 1205, the perturbed cable position by the curves 1210, 1215, and the position after inversion by the curves 1220, 1225. The errors (E_x, E_y, E_z) in the position of each sensor and the square sum ($E_{xyz} = sqrt(E_x^2 + E_y^2 + E_z^2)$) of the linearised solution are shown in **FIG. 13**. On average, E_{xyz} = 14 m for one acoustic source with no dip measurement. For the second simulation, three acoustic sources in a triangular shape were positioned at (-12.5, 0, 0), (37.5, 70, 0), (37.5, -50, 0). Plots of the cable position are shown in **FIG. 14**. The errors (E_x, E_y, E_z) in the position of each sensor are shown in **FIG. 15**. The additional two acoustic sources reduces the error particular in the crossline direction to E_{xyz}=4 m - 10 m. For the third experiment, dip angle measurements along the cable and the three acoustic sources were included. Plots of the cable position are shown in **FIG. 16.** The errors (E_x, E_y, E_z) in the position of each sensor are shown in **FIG. 17**. The dip measurements further reduce the error, particular in the *z*-coordinate, to E_{xyz} = 2 m - 4 m.

Note that, although the illustrated embodiment pertains to a seismic cable deployed in water, the invention is not so limited and may be employed in other contexts. The method of the invention can be extended to the deployment of any cable in any medium, or even in a vacuum. Note that there is no such thing as a true vacuum, and even environments considered a "vacuum" contain some form of medium. Some of these variations may affect the availability, desirability, or selection of constraints that may be used in determining the positions from the angular measurements. Similarly, the invention may be applied to stationary cables, *e.g.*, a cable previously deployed and at rest on the sea floor. In these embodiments, the angular orientation measurement and the determination need not be made dynamically and the determination need not be made on the fly. These and other variations on the embodiments disclosed herein will become apparent to those skilled in the art having the benefit of this disclosure.

Thus, the invention uses, in the illustrated embodiment, combination of angular measurements made along the length of a cable with other constraints such as, acoustic arrival times between acoustic sources and acoustic sensors also mounted on the cable, to determine the cable's shape in real-time. The angular measurements provide additional constraints that can improve the accuracy of a solution based on acoustic measurements only. It may also be possible to reduce the complexity of the acoustic system through the use of additional information afforded by measurements of angle. In the illustrated embodiment, the technique disclosed herein is implemented such that cable position can be determined within 2 – 3 seconds of measurements being taken, in pseudo real-time. This permits one to effect control of the seismic cable's shape in the catenary as it is being deployed. This real-time, or pseudo real-time, cable position determination (including uncertainties) lends itself to an iterative process.

The various embodiments and aspects of the invention disclosed herein consequently include a method and an apparatus for determining a position of an acoustic receiver. The apparatus includes at least one acoustic source; an acoustic receiver, and a computing system. The acoustic receiver is capable of receiving a plurality of acoustic signals transmitted by the at least one acoustic source from at least two signal source positions. The computing system is programmed to determine a position of the acoustic receiver from the acoustic ranges between the at least two signal source positions and the acoustic receiver and a non-acoustic

constraint. The method includes determining a first and a second acoustic range from a first signal source position and a second signal source position, respectively, to the acoustic receiver; ascertaining a non-acoustic constraint on the acoustic receiver's position; and determining the acoustic receiver's position from the first and second acoustic ranges and the non-acoustic constraint.

This concludes the detailed description. The particular embodiments disclosed above are illustrative only, as the invention may be modified and practiced in different but equivalent manners apparent to those skilled in the art having the benefit of the teachings herein. Furthermore, no limitations are intended to the details of construction or design herein shown, other than as described in the claims below. It is therefore evident that the particular embodiments disclosed above may be altered or modified and all such variations are considered within the invention as claimed below. Accordingly, the protection sought herein is as set forth in the following claims.

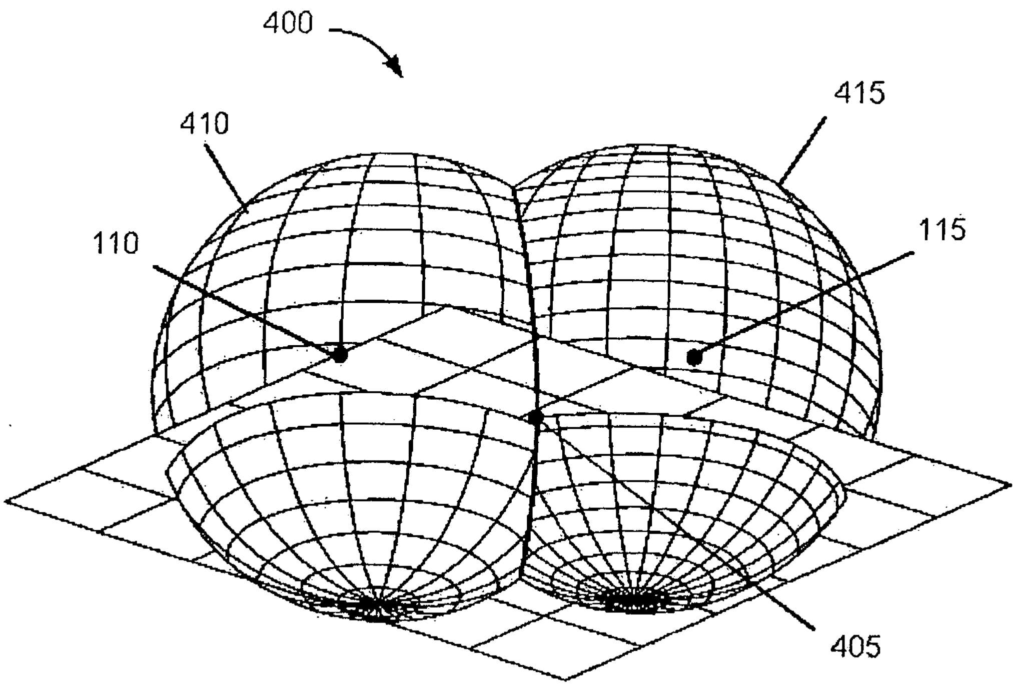


FIG. 4

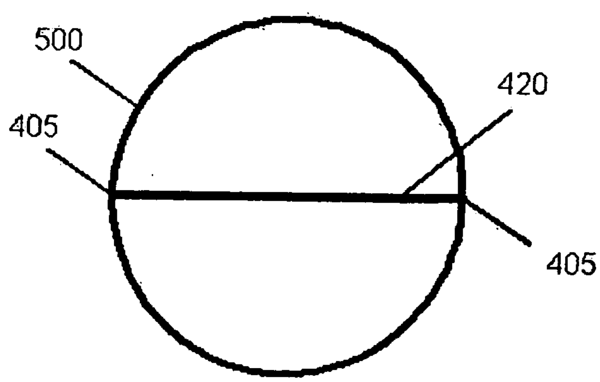


FIG 5

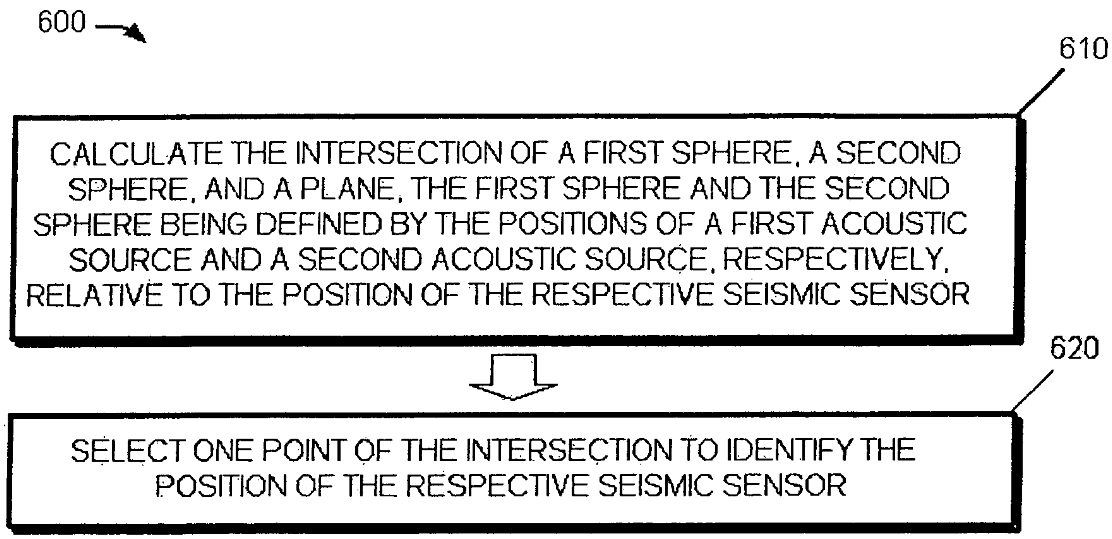

600
610
CALCULATE THE INTERSECTION OF A FIRST SPHERE, A SECOND SPHERE, AND A PLANE, THE FIRST SPHERE AND THE SECOND SPHERE BEING DEFINED BY THE POSITIONS OF A FIRST ACOUSTIC SOURCE AND A SECOND ACOUSTIC SOURCE, RESPECTIVELY, RELATIVE TO THE POSITION OF THE RESPECTIVE SEISMIC SENSOR
620
SELECT ONE POINT OF THE INTERSECTION TO IDENTIFY THE POSITION OF THE RESPECTIVE SEISMIC SENSOR

FIG. 6

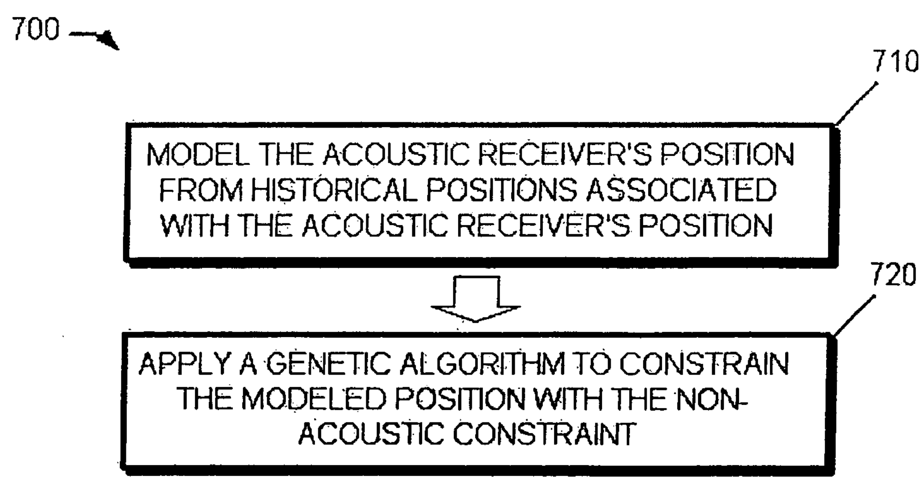
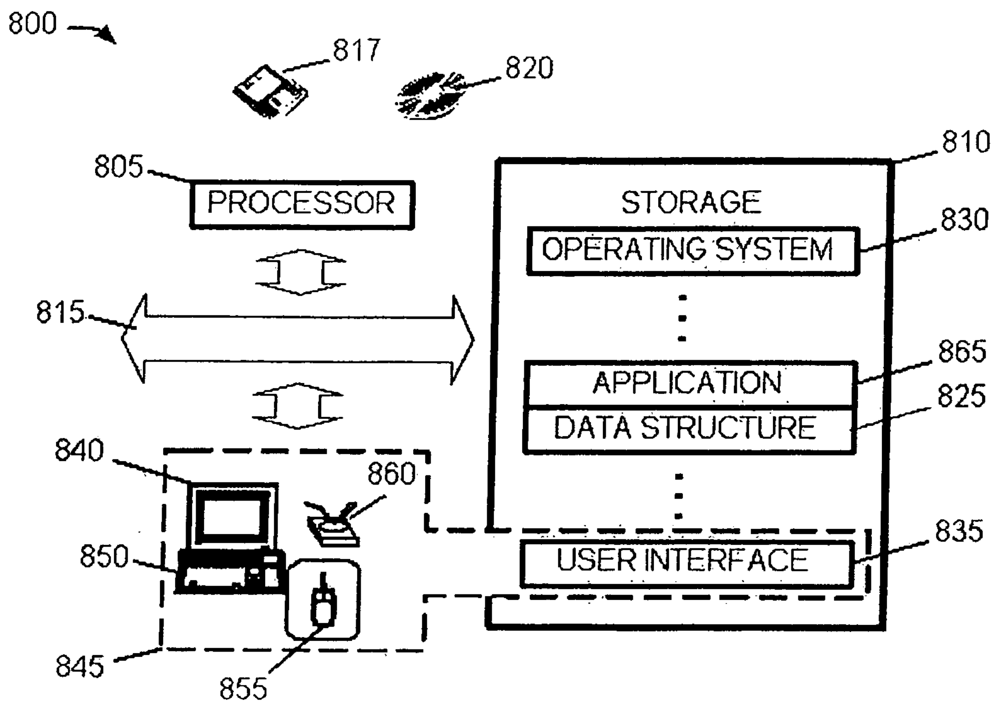
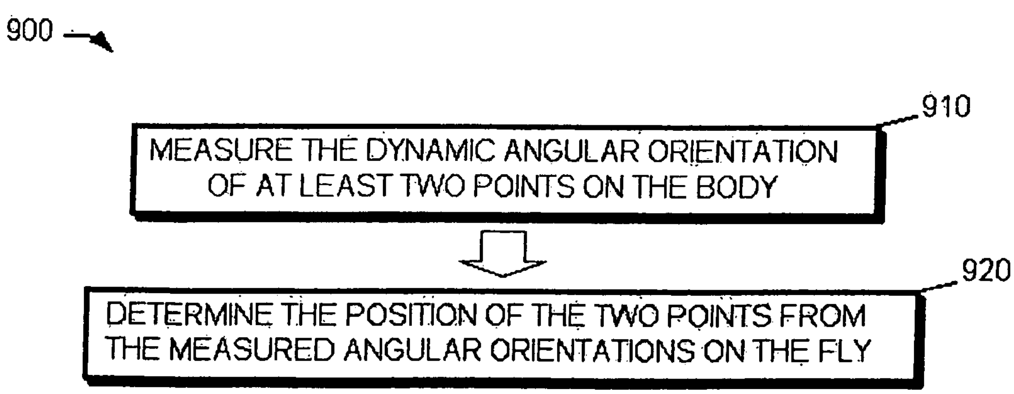

700
710
MODEL THE ACOUSTIC RECEIVER'S POSITION FROM HISTORICAL POSITIONS ASSOCIATED WITH THE ACOUSTIC RECEIVER'S POSITION
720
APPLY A GENETIC ALGORITHM TO CONSTRAIN THE MODELED POSITION WITH THE NON-ACOUSTIC CONSTRAINT

FIG 7



800
817
820
810
805
PROCESSOR
STORAGE
830
OPERATING SYSTEM
815
865
APPLICATION
825
DATA STRUCTURE
840
860
835
USER INTERFACE
850
845
855

FIG. 8



900
910
MEASURE THE DYNAMIC ANGULAR ORIENTATION OF AT LEAST TWO POINTS ON THE BODY
920
DETERMINE THE POSITION OF THE TWO POINTS FROM THE MEASURED ANGULAR ORIENTATIONS ON THE FLY

FIG. 9

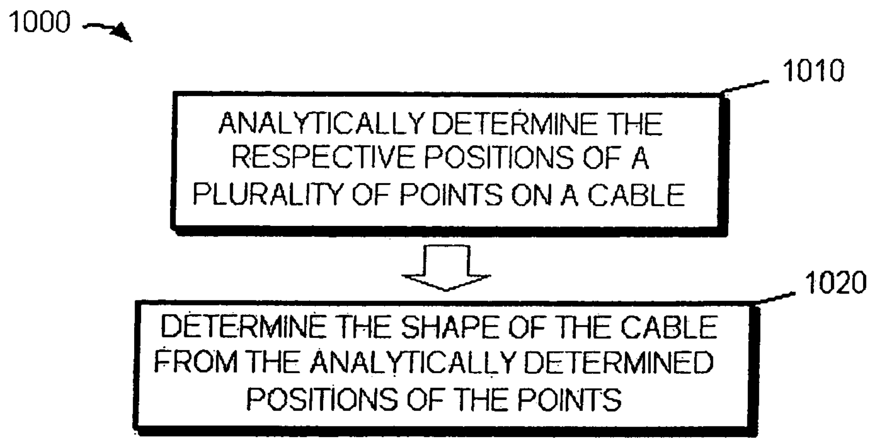

1000
1010
ANALYTICALLY DETERMINE THE RESPECTIVE POSITIONS OF A PLURALITY OF POINTS ON A CABLE
1020
DETERMINE THE SHAPE OF THE CABLE FROM THE ANALYTICALLY DETERMINED POSITIONS OF THE POINTS

FIG. 10

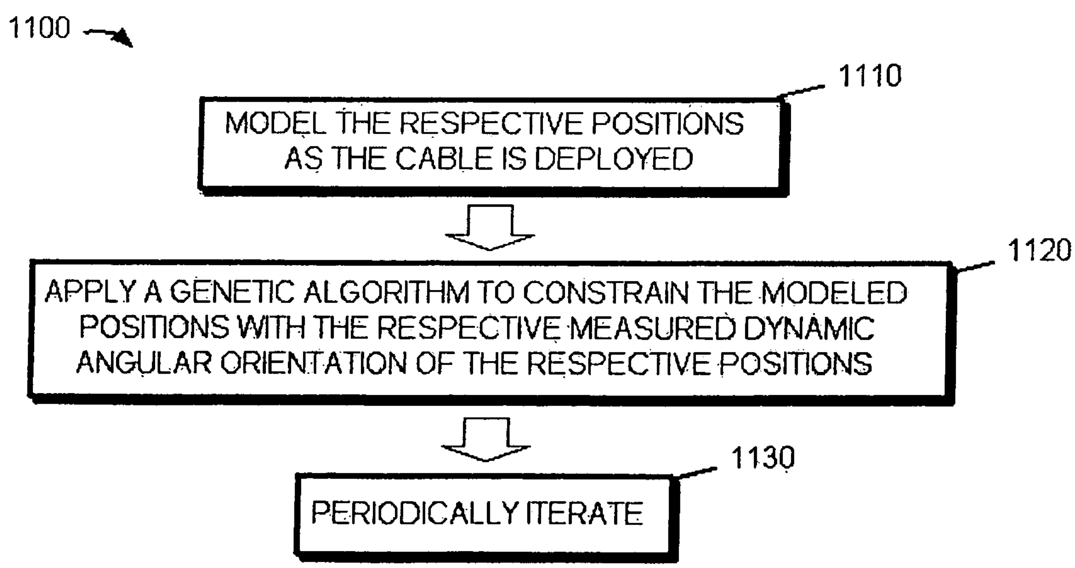

1100
1110
MODEL THE RESPECTIVE POSITIONS AS THE CABLE IS DEPLOYED
1120
APPLY A GENETIC ALGORITHM TO CONSTRAIN THE MODELED POSITIONS WITH THE RESPECTIVE MEASURED DYNAMIC ANGULAR ORIENTATION OF THE RESPECTIVE POSITIONS
1130
PERIODICALLY ITERATE

FIG. 11

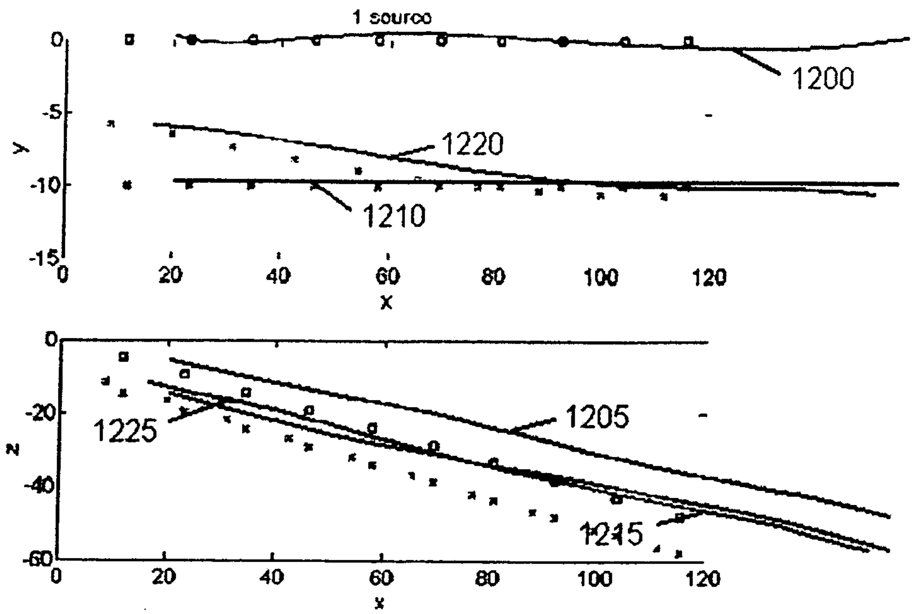

1 source
0
-5
-10
-15
y
0
20
40
60
80
100
120
x
1200
1220
1210
0
-20
-40
-60
z
1205
1225
1215

FIG. 12

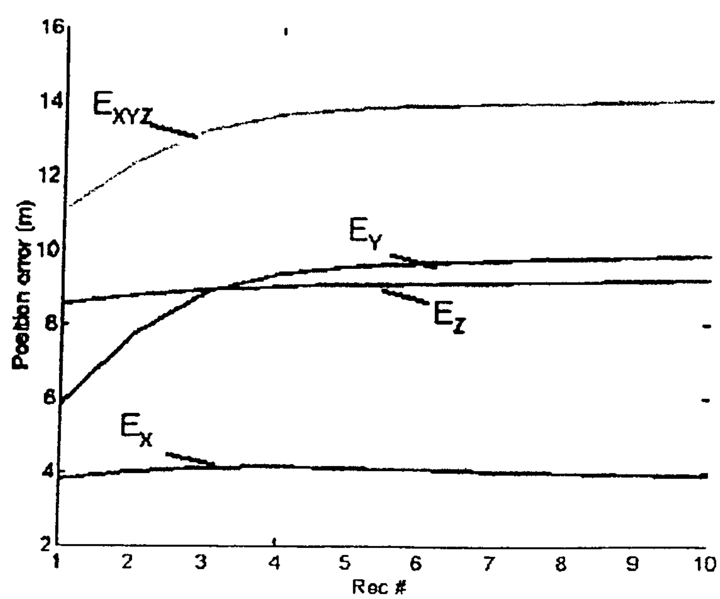
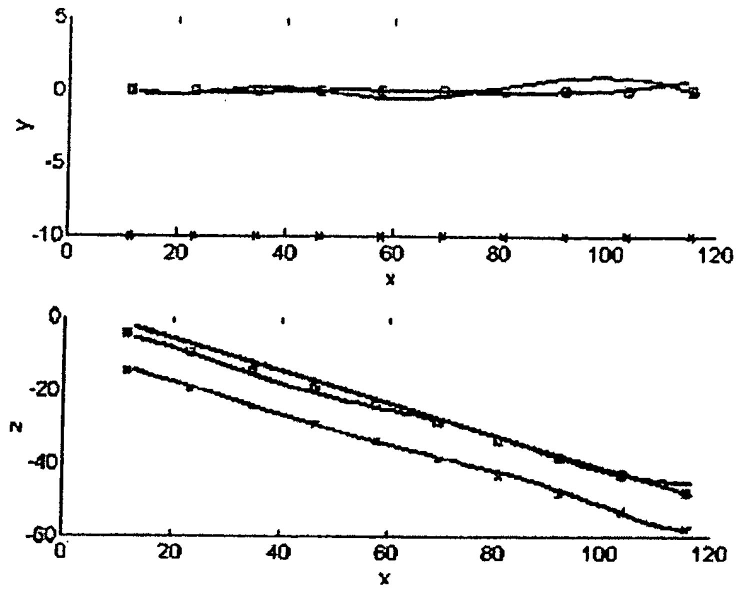
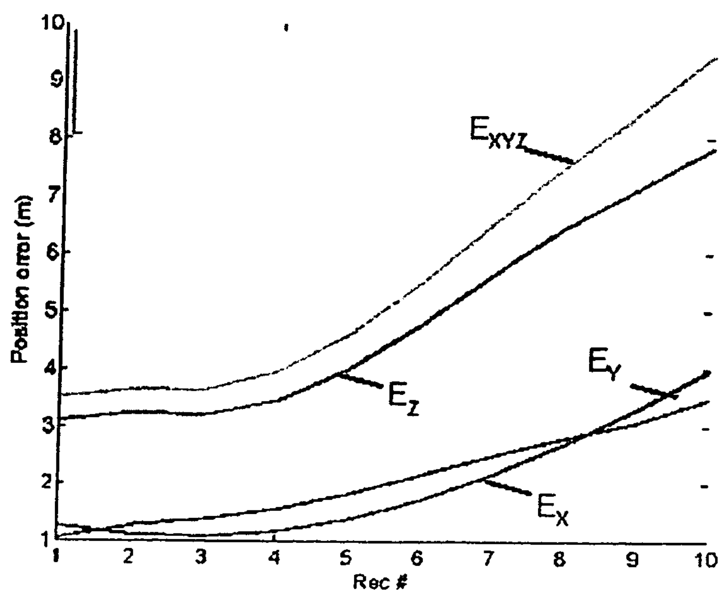

16
14
12
10
8
6
4
2
Position error (m)
1
2
3
4
5
6
7
8
9
10
Rec #
E_{XYZ}
E_Y
E_Z
E_X

FIG 13



5
0
-5
-10
y
0
20
40
60
80
100
120
x
0
-20
-40
-60
z
x

FIG. 14



Position error (m)
Rec #
E_{XYZ}
E_Z
E_Y
E_X

FIG. 15

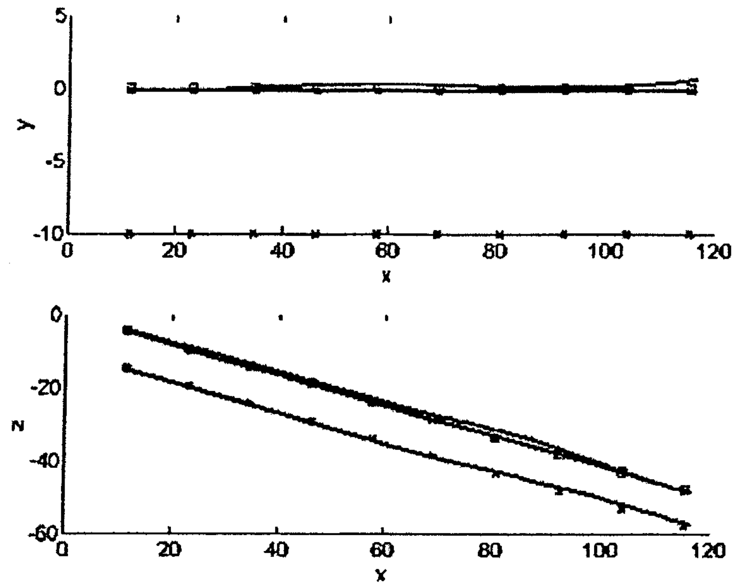


FIG. 16

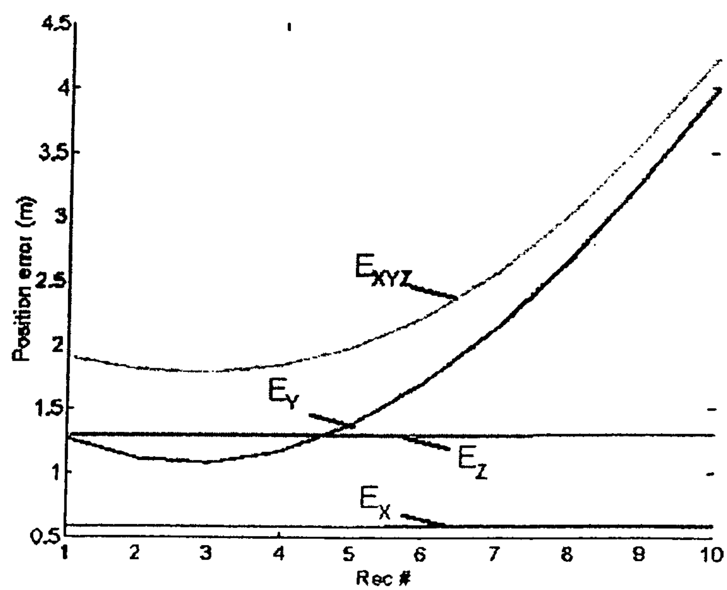


FIG. 17